

# Application of Queue Model for Performance Assessment of Multi-Channel Multi-Servers Motor Spirit Filling Station

B.O. Akinnuli, T.O. Olugbade.

Department of Mechanical Engineering, Federal University of Technology, P.M.B.704, Akure, Nigeria

*Abstract - Queuing analysis is to offer a reasonably satisfactory service to waiting customers. It is not an optimization technique. It determines the measures of performance of waiting lines such as the average waiting time in queue and the productivity of the service facility; this can be used to design the required service installation. Queuing models vary from single to multiple channel systems of arrangement. The rate of arrival of customers requiring service a times is greater than the rate of service, this imbalance may be temporal but during that temporary imbalance period, a queue is always formed. Formation of this queue was found to cause increase of customers waiting time, overstressing of the available servers and loss of goodwill in this case study. Hence the adoption and application of multi-channels and parallel multi-servers model for proffering solution to this filling station bottleneck. The arrival of customers in this case study, followed a poisson probability distribution at an average rate of customers arrival per unit of time and are served on a first-come, first-served basis by any of the servers. The services times are distributed exponentially. The performance measure of this adopted queue model was carried out to be able to know numbers of customers waiting in the queue and in the system, waiting time of the customers in the queue and in the system as well as the utilization factor for the servers in this case study. Based on this, management decision was taken from the revelation of the performance assessment of this filling station and policy for running this filling station was formulated. Application of this adopted queue model by using data collected from Nigeria National Petroleum Corporation (NNPC) Mega Station revealed that as number of servers increases, customers' average arrival rate decreases; increase in average service rate reduces the average waiting time in this system; as average customer in the system reduces so also average customers in the queue. Also as the probability of the system being busy reduced, the probability of idleness increases. The case study is using 7 attendants presently, which gave utilization factor of 32% bringing the idleness probability to 68%. Though service rate increases while the system and queue time reduces; queue formed in the system was negligible. The calculated mean for the servers' utilization factor is 0.46 (46%) as concerned this study. The 0.47 (47%) obtained is closest to this value and hence selected as the minimum benchmark. This minimum benchmark of 47% was to select the number of attendant required for this system. At utilization of 0.682 and idleness of 0.292, 3 attendants will be required. The waiting time in the system  $W_s = 0.20\text{hr}$ , waiting time on the queue  $W_q = 0.13\text{hr}$ , average customers' queue  $L_q = 1$ , service rate  $\mu = 15$  with arrival rate of 10 customers per hour. The selected number of attendant is five (5). Incentives can be given to create overtime that will increase the utilization factor of these five*

*attendance. Any value below this benchmark is not encouraged for this system.*

**Keywords:** Queue, model, performance assessment, multi-channel, multi-servers, utilization factor.

## I. INTRODUCTION

The queuing problem is a problem about a balance between average waiting time of customers and the idle time of the attendants in the filling station [12]. Queuing theory is a collection of mathematical models of various queuing systems. It is used extensively to analyze production and service processes exhibiting random variability in market demand (arrival times) and service times [4]. They went further to say that the problem of balancing the cost of waiting against the cost of idle time of service facilities in the system arises due to the probabilistic nature of the inter-arrival times of customers and the time taken to complete the service to the customers. Numerous investigators have studied sequential queuing systems in filling station and their researches have contributed greatly to knowledge as far as operations analysis of a filling station is concerned. According to [1], a case of a single server where multiple customers on a single queue are being attended to by a single machine attendant can be used. He used simulation to find the mean and the variance of the time in the system in terms of the busy time of the machine attendant and the delayed time of the customers on queue. According to [11] and [2], the problem of two queues in parallel, with independent poisson arrival process and general renewal service time processes attended by a single machine attendant who switches without delay from one queue to the other when the present queue is empty can be solved. He obtains the Laplace transform of the waiting lines in each queue in the filling station. The case of random length of switching times is studied by [3] focusing on two queues with change over times in a filling station. He considers two switching rules (switch when present queue is empty, and the case in which one of the queues has non-preemptive priority over the other and finds the Laplace transform of the waiting lines of the customers in the filling station. They all worked on a single server with one or two channels. This research focuses on multiple servers and multiple queues in a filling station using Nigeria National Petroleum Corporation (NNPC) Mega Filling Station at Akure,

Ondo State, as case study. Some prominent models used in the queue problems and their developers are stated in the table below.

**Table 1. Mathematical Models Identified**

S/N	Queue Component	Mathematical Model	Author(s)	Year
1.	Entity Arrival Model	$P \{N(t+s) - N(t) = K\} = e^{-\lambda s} (\lambda s)^k / k!$ ----- $t \geq 0, s \geq 0, k = 0, 1, 2, \dots$	[12]	2002
2.	Service Model	$F(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} - \infty < \mu < \infty, \sigma > 0$	[12]	2002
3.	Mean Delaying Time	$d = \lim_{n \rightarrow \infty} \sum_{i=1}^n Di/n$	[12]	2002
4.	Mean Staying Time	$W = \lim_{n \rightarrow \infty} \sum_{i=1}^n (wi/n) = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{(Di+Si)}{n}$	[5]	2002
5.	Mean Step Length	$Q = \lim_{T \rightarrow \infty} \frac{\int_0^T Q(t)dt}{T}$	[12]	2002
6.	Mean Entity Number	$L = \lim_{T \rightarrow \infty} \frac{\int_0^T L(t)dt}{T} = \lim_{T \rightarrow \infty} \frac{\int_0^T Q(t)+S(t)dt}{T}$	[5]	2002
7.	Average Customer (system). Single Channel	$Ls = \frac{\lambda}{\mu - \lambda}$	[10]	2004
8.	Average Waiting Time (system). Single channel	$Ws = \frac{1}{\mu - \lambda}$	[10]	2004
9.	Average Customer (Queue). Single channel	$Lq = \frac{\lambda^2}{\mu(\mu - \lambda)}$	[10]	2004
10.	Average Waiting Time (Queue). Single channel	$Wq = \frac{\lambda}{\mu(\mu - \lambda)}$	[10]	2004
11.	Probability of System Busy. Single channel	$P = \frac{\lambda}{\mu}$	[10]	2004
12.	Probability of System Idle. Single channel	$Po = 1 - \frac{\lambda}{\mu}$	[10]	2004
13.	Probability of more k customers	$P(n > k) = (\lambda/\mu)^{k+1}$	[10]	2004

This research is no doubt controls the overall production capabilities of most filling stations with multi-channels and multi-servers system of operations to predict the number of attendants required, the queues formed or allowed for customers waiting for service. In addition to this, this research creates simulation techniques opportunity for solving bottlenecks in multi-channels and multi-servers filling station. This paper addresses the issue of how queues formed in the system and how they can be reduced to the minimum. It predicts the number of servers that will be needed in the system for cost minimization and profit optimization. However, multi-channels multi-servers system in which customers on different queues are attended to by multiple servers was only considered in this research as the queue configuration.

## II. METHODOLOGY

The procedure for the required model development, performance measures and stability of the model were considered here.

### A. Model Development

In this case instead of single server, there are multiple but identical servers in parallel to handle servicing customers. The arrival pattern was assumed to follow poisson probability distribution at an average rate of  $\lambda$  customers per unit of time and is served on a first-come, first-served basis by any of the servers. The service times are distributed exponentially with an average of  $\mu$  customers per unit of time. It is further assumed that only one queue is formed. If there are  $n$  customers in the queuing system at any point in time, the following two cases may arise:

(i) If  $n < s$ , (number of customers in the system is less than the number of servers) then there will be no queue. However,  $(s-n)$  numbers of servers are not busy. The combined service rate will then be:

$$\mu_n = n\mu; n < s \quad (1)$$

(ii) If  $n \geq s$ , (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be  $(n - s)$ . The combined service rate will be:

$$\mu_n = S\mu; n \geq s \quad (2)$$

Thus to derive the results for this model, we have

$\mu_n = \lambda$  for all  $n \geq 0$

$$\mu_n = \begin{cases} n\mu; & n < s \\ S\mu, & n \geq s \end{cases} \quad (3)$$

The method of determining probability,  $P_n$  of  $n$  customers in the queuing system at time  $t$  and value of its various characteristics is as shown thus:

### 1. Determination of Differential-Difference Equations of the System

$$P_n(t + \Delta t) = P_n(t)\{1 - \lambda\Delta t\}\{1 - n\mu\Delta t\} + P_{n+1}(t)\{1 - \lambda\Delta t\}\{(n + 1)\mu\Delta t\} + P_{n-1}(t)\{\lambda\Delta t\}\{1 - (n - 1)\mu\Delta t\} \\ = -(\lambda + n\mu)P_n(t)\Delta t + (n + 1)\mu P_{n+1}(t)\Delta t + \lambda P_{n-1}(t) + P_n(t) + \text{terms involving } (\Delta t)^2; 1 \leq n < s \quad (4)$$

$$P_n(t + \Delta t) = P_n(t)\{1 - \lambda\Delta t\}\{1 - \mu\Delta t\} + P_{n+1}(t)\{1 - \lambda\Delta t\}\{s\mu\Delta t\} + P_{n-1}(t)\{\lambda\Delta t\}\{1 - s\mu\Delta t\} \\ = -(\lambda + s\mu)P_n(t)\Delta t + s\mu P_{n+1}(t)\Delta t + \lambda P_{n-1}(t) + P_n(t) + \text{terms involving } (\Delta t)^2; n \geq s \quad (5)$$

and

$$P_0(t + \Delta t) = P_0(t)(1 - \lambda\Delta t) +$$

$$P_1(t)\mu\Delta t; n = 0$$

[8]

Dividing these equations by  $\Delta t$  and then taking limit as

$\Delta t \rightarrow 0$ , we get

$$P'_n(t) = -(\lambda + n\mu)P_n(t) + (n + 1)\mu P_{n+1}(t) + \lambda P_{n-1}(t); 1 \leq n < s$$

$$P'_n(t) = -(\lambda + s\mu)P_n(t) + (s\mu P_{n+1}(t) + \lambda P_{n-1}(t)); n \geq s$$

and

$$P'_0(t) = -(\lambda P_0(t) + \mu P_1(t)); n = 0$$

$$0 \quad (6)$$

### 2. Determination of the Steady-State Equations of the System

In the steady-state condition, the differential equations are obtained from the above equations as  $t \rightarrow \infty$ . This yield

$$-\lambda P_0 + \mu P_1 = 0; \quad n = 0 \\ -(\lambda + n\mu)P_n +$$

(n

$$+ 1)\mu P_{n+1} + \lambda P_{n-1} = 0; \quad 0 < n <$$

s

$$-(\lambda + s\mu)P_n + s\mu P_{n+1} + \lambda P_{n-1} = 0; \quad n \geq s$$

### 3. Solving Difference Equations of the System

By applying the iterative method, the probability of  $n$  customers in the system is given by

$P_n$

$$= \begin{cases} \frac{\rho^n}{n!} P_0 & ; n \leq s \\ \frac{\rho^n}{s! s^{n-s}} P_0 & ; n > s; \rho = \lambda/s\mu \end{cases} \quad (7)$$

In order to find  $P_0$ , we use the condition

$$\begin{aligned}
 1 &= \sum_{n=0}^{\infty} P_n = \sum_{n=0}^{s-1} P_n + \sum_{n=s}^{\infty} P_n = \sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n P_0 + \sum_{n=s}^{\infty} \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^n P_0 \\
 &= P_0 \left[ \sum_{n=0}^{s-1} \frac{s^n}{n!} \left(\frac{\lambda}{s\mu}\right)^n + \sum_{n=s}^{\infty} \frac{s^n}{s! s^{n-s}} \left(\frac{\lambda}{s\mu}\right)^n P_0 \right] \\
 &= P_0 \left[ \sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{s^s}{s!} \sum_{n=s}^{\infty} \rho^n \right] = P_0 \left[ \sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{s^s \rho^s}{s! (1-\rho)} \right]; \rho = \frac{\lambda}{s\mu}
 \end{aligned}$$

$$P_0 = \left[ \sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{s^s \rho^s}{s! (1-\rho)} \right]^{-1}; \rho = \frac{\lambda}{s\mu}$$

$$\left[ \text{since } \sum_{n=s}^{\infty} \rho^n = \rho^s + \rho^{s+1} + \dots = \rho^s / (1-\rho), \text{ sum of infinite G.P. ; } \rho < 1 \right]$$

Thus the probability that the system shall be idle is

$$\begin{aligned}
 P_0 &= \left[ \sum_{n=0}^{s-1} \frac{(s\rho)^n}{n!} + \frac{1}{s!} \frac{(s\rho)^s}{1-\rho} \right]^{-1}; \rho = \lambda/s\mu \\
 &= \left[ \sum_{n=0}^{s-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{s\mu}{s\mu - \lambda} \right]^{-1}
 \end{aligned}$$

**B. Performance Measures for the Model**

The performance measures considered involved: expected number of customers waiting in the queue and in the system, expected waiting time of the customer in the queue and in the system as well as the probability that all servers are simultaneously busy (utilization factor)

**1. Model for Expected Number of Customers Waiting in the Queue**

$$L_q = \sum_{n=s}^{\infty} (n-s) P_n = \sum_{n=s}^{\infty} (n-s) P_n \frac{\rho^n}{s! s^{n-s}} P_0 \tag{9}$$

$$\begin{aligned}
 &= \frac{\rho^s}{s!} P_0 \sum_{n=s}^{\infty} (n-s) \rho^{n-s}; \rho = \frac{\lambda}{\mu} \\
 &= \frac{\rho^s}{s!} P_0 \sum_{m=0}^{\infty} m \rho^m; n-s = m \\
 &= \frac{\rho^s}{s!} \rho P_0 \sum_{m=0}^{\infty} m \rho^{m-1} = \frac{\rho^s}{s!} \cdot \rho P_0 \frac{d}{d\rho} \left[ \sum_{m=1}^{\infty} \rho^m \right] \\
 &= \frac{\rho^s}{s!} \rho P_0 \frac{1}{(1-\rho)^2} = \left[ \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s \frac{\lambda s\mu}{(s\mu - \lambda)^2} \right] P_0 \\
 &= \left[ \frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\lambda\mu}{(s\mu - \lambda)^2} \right] P_0
 \end{aligned}$$

**2. Model for Expected Number of Customers in the System**

$$L_s = \left[ L_q + \frac{\lambda}{\mu} \right] \tag{11}$$

**3. Model for Expected Waiting Time of a Customer in the Queue**

$$W_q = \left[ \frac{1}{(s-1)!} \left(\frac{\lambda}{\mu}\right)^s \frac{\mu}{(s\mu - \lambda)^2} \right] P_0 = \frac{L_q}{\lambda} \tag{12}$$

**4. Model for Waiting Time that a Customer Spends in the System**

$$W_s = W_q + \frac{1}{\mu} \tag{10}$$

**5. Model for Probability that all Servers are simultaneously Busy (Utilization factor)**

$$\begin{aligned}
 P(n \geq s) &= \sum_{n=s}^{\infty} P_n = \sum_{n=s}^{\infty} \frac{1}{s!} \frac{1}{s^{n-s}} \left(\frac{\lambda}{\mu}\right)^n P_0 \\
 &= \frac{1}{s!} \left(\frac{\lambda}{\mu}\right)^s P_0 \sum_{m=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^m \frac{s\mu}{s\mu - \lambda} P_0
 \end{aligned} \tag{14}$$

**C. The Stability System for Multi-Channel Multi-Server System**

The stability system for multi-channel multi-server system was studied and the stability equation given as:

$$\sum E[A_i(t)] < K - \sum_{k=1}^K \prod_{i=Q} (1 - P_k) \forall Q \subset \{1, 2, \dots, L\} \tag{15}$$

Where,

A<sub>i</sub> = number of arrivals at channel i,

K = number of servers

L = number of channels

Equation (13) above explains the stability system for multi-channel multi-server system in which A<sub>i</sub> represents the number of arrivals at channel i at a particular time, K represents all the number of servers in the system and L represents all the number of channels in the system. There will be N customers reaching the system at different or the same time, and the interval of customers reaching the system and the time of accepting service are random. Customers select the shortest queue to stand in after reaching the system. If all the queues are of the same length, customers enter according to the rate and movement of the queue. Consider a discrete time single queue system with an arrival process A(t) and service process μ(t). Assume that the arrivals are added to the system at the end of each time slot. Hence, the queue length process X(t) evolves with time according to the following rule [9].

$$X(t) = (X(t-1) - \mu(t)) + A(t) \tag{10}$$

A queue satisfying the conditions above is called strongly stable if

$$\lim_{t \rightarrow \infty} \sup \frac{1}{t} \sum_{T=0}^{t-1} E[X(T)] < \infty$$

(17) [7]

Customers form a queue in order of arrival into the system and leave the system as soon as they receive service as shown in fig.1 below.

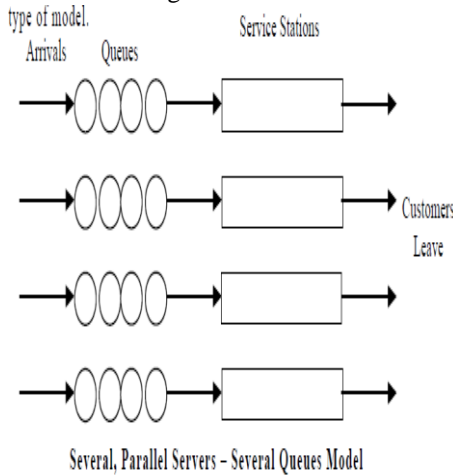


Fig.1 several parallel Servers –Several Queues Model.

Source: [6]

The major components of queuing system are the arrival rate of the customer, queue or waiting-line formed, the service rate and the outlet. Therefore, mathematical model for the entity arrival mode, service mode and the criteria of the queuing system are given below:

- $\lambda$  = mean number of arrivals per time period
- $\mu$  = mean number of people or items served per time period
- $k$  = number of servers in the system

To run this software efficiently, the following requirements must be met:

- Operating System: Window XP Professional
- Processor Speed: 720MHz and higher processors
- RAM: 572MB at least
- Hard Disk Space: 2GB at least

The coding environment as displayed above gives room for proper arrangement of codes, instructions and commands

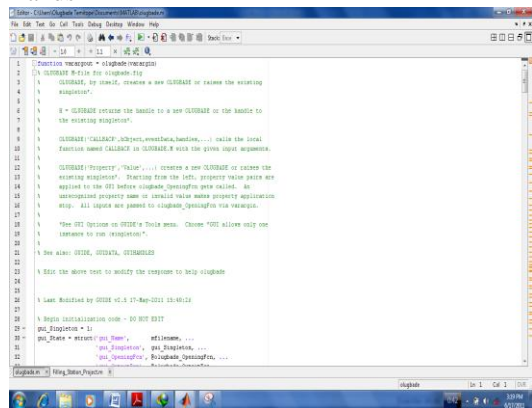
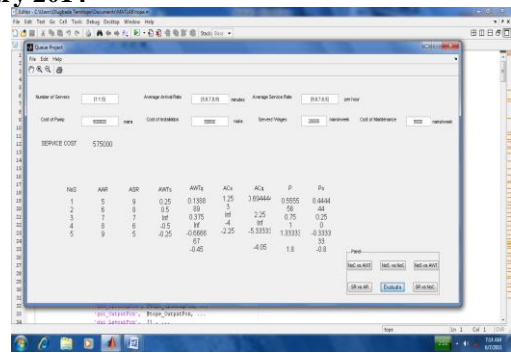


Fig.3 Coding Environment



NO.	ARR	SER	AVTS	AVTS	ACs	ACs	P	Pv
1	5	9	0.25	0.1908	1.25	1.0444	0.9555	0.4444
2	5	9	0.15	0.08	3	1.0444	0.9555	0.4444
3	7	7	0.25	0.375	0.75	1.0444	0.9555	0.4444
4	5	9	0.25	0.1908	1.25	1.0444	0.9555	0.4444
5	9	5	0.25	0.6667	2.25	1.0444	0.9555	0.4444

Fig. 4. Results Sheet displaying the generated results from the Software

#### D. Determination of the Bench mark for the number of the attendants required.

The mean for the entire utilization factor is to be determined. Any value below the mean value is hereby rejected, because utilization factor above the mean (bench mark) caused increment of idle time, less production time and increase in service rate.

$$\text{The mean: } X = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$$= \frac{0.92 + 0.79 + 0.67 + \dots + 0.04}{12} = \frac{5.52}{12}$$

$$= 0.46$$

From the result table, the closer utilization value is 0.47. Based on the utilization factor of 0.47, the number of attendants required for this system is 5 attendants (since the use of 5 attendants will make the utilization factor to be) Therefore to determine the acceptable utilization factor for the policy making in this system requires the average value for the utilization factor between the highest value of 0.95 to the bench mark value of 0.47.

This gives

$$x = \frac{0.92 + 0.79 + 0.67 + 0.56 + 0.47}{5} = 0.682$$

$$P_{av} = 0.682$$

By reason of interpolation, the idleness percentage is determined

$$\frac{0.92 - 0.682}{0.682 - 0.47} = \frac{0.53 - x}{x - 0.08}$$

$$0.238(x - 0.08) = 0.212(0.53 - x)$$

$$0.45x = 0.1314$$

$$x = 0.292 = P_0$$

$$\therefore P = 0.682 \text{ and } P_0 = 0.292$$

### III. RESULTS AND DISCUSSION

The results obtained from this model showed that too many servers lead to excess expenses on salary and equipment as well as idleness of servers while too few servers lead to over stressing of servers which kill their goals and make customers wait too long on queue. This can cause loss of goodwill. Therefore, the number of servers must be optimized.

**Table 2: Summary of the Simulation Result**

No of Servers (k)	Average Arrival Rate, $\lambda$ (hrs)	Average Service Rate, $\mu$ (hrs)	Average Waiting Time (system), $W_s$ (hrs)	Average Waiting Time (Queue), $W_q$ (hrs)	Average Customer (system), $L_s$	Average Customer (Queue), $L_q$	Prob. of System Busy, P (%)	Prob. of System Idle, Po (%)
1	5	7	0.50	0.38	2.50	1.79	71.42	28.57
2	4	8	0.25	0.13	1.00	0.50	50.00	50.00
3	3	9	0.17	0.06	0.50	0.17	33.33	66.67
4	2	10	0.13	0.03	0.25	0.05	20.00	80.00
5	1	11	0.10	0.01	0.10	0.01	09.09	90.91

Table 2 is the summary of the numerical results generated from the software. The average waiting time of customer in the system was 0.1 hrs for 5 servers, 0.17 hrs for 3 servers, 0.25 hrs for 2 servers and 0.5 hrs for 1 server. From the customer scheduling policy models identified, it was noted that the average waiting time of customer in the queue and in the system is a function of the service rate of the servers. As the number of servers in the system increases, the average customers in the system waiting for service reduces which reduce the waiting line or queue formed.

**IV. CONCLUSION**

This research controls the overall production capabilities of most filling stations with multi-channels and multi-servers system of operations to predict the number of attendants required, the queues formed or allowed for customers waiting for service. In addition to this, this research creates simulation techniques opportunity for solving bottlenecks in multi-channels and multi-servers filling station. The issue of how queues formed in the system was addressed and how they can be reduced to the minimum was also discussed. This helps in predicting the number of servers that will be needed in the system for cost minimization and profit optimization.

**REFERENCES**

[1] A.A. Abdul: "A Feasibility Study on the Proposed Petrol Station on Students' Campus of Higher Learning (Case Study: University Technology Malaysia, Skudai), Feb. 2003.

[2] H. Al-Zubaidy, L. Lambadaris and Viniotis: "Optimal resource scheduling in wireless multi service systems with random channels connectivity", in proc. Of IEEC Global Communication Conference, Nov.2009.

[3] L. Anselin: Under the Hood. Issues in the Specification and Interpretation of Spatial Regression Models. In: Regional Economics Applications Laboratory (REAL), Nov. 2002.

[4] S. Bodas, S. Shakkottai, L. Ying and R. Srikant: "Scheduling in multi-channel wireless networks: Rate function optimality in the small-buffer regime. Technical report, The University of Texas at Austin, WNCG, Jan. 2009.

[5] E. Boyle: "The Failure of Business Format Franchising in British Forecourt Retailing: A Case Study of the Rebranding of Shell Retail's Forecourts. International Journal of Retail & Distribution Management, June, 2002.

[6] H. Halabian, I. Lambadaris and C.H. Lung: "Network capacity region of multi-queue multi-server queuing system with time varying connectivity's," in Proc. of IEEE Int. Symp. On Inform. Theory (ISIT'10), Austin, TX, USA, June 2010.

[7] H. Halabian, I. Lambadaris and C.H. Lung: "On the Stability Region of Multi-Queue Multi-Server Queuing Systems with Stationary Channel Distribution. 2011 IEEE, International Symposium on Information Theory Proceedings: pp.1709, Feb. 2011.

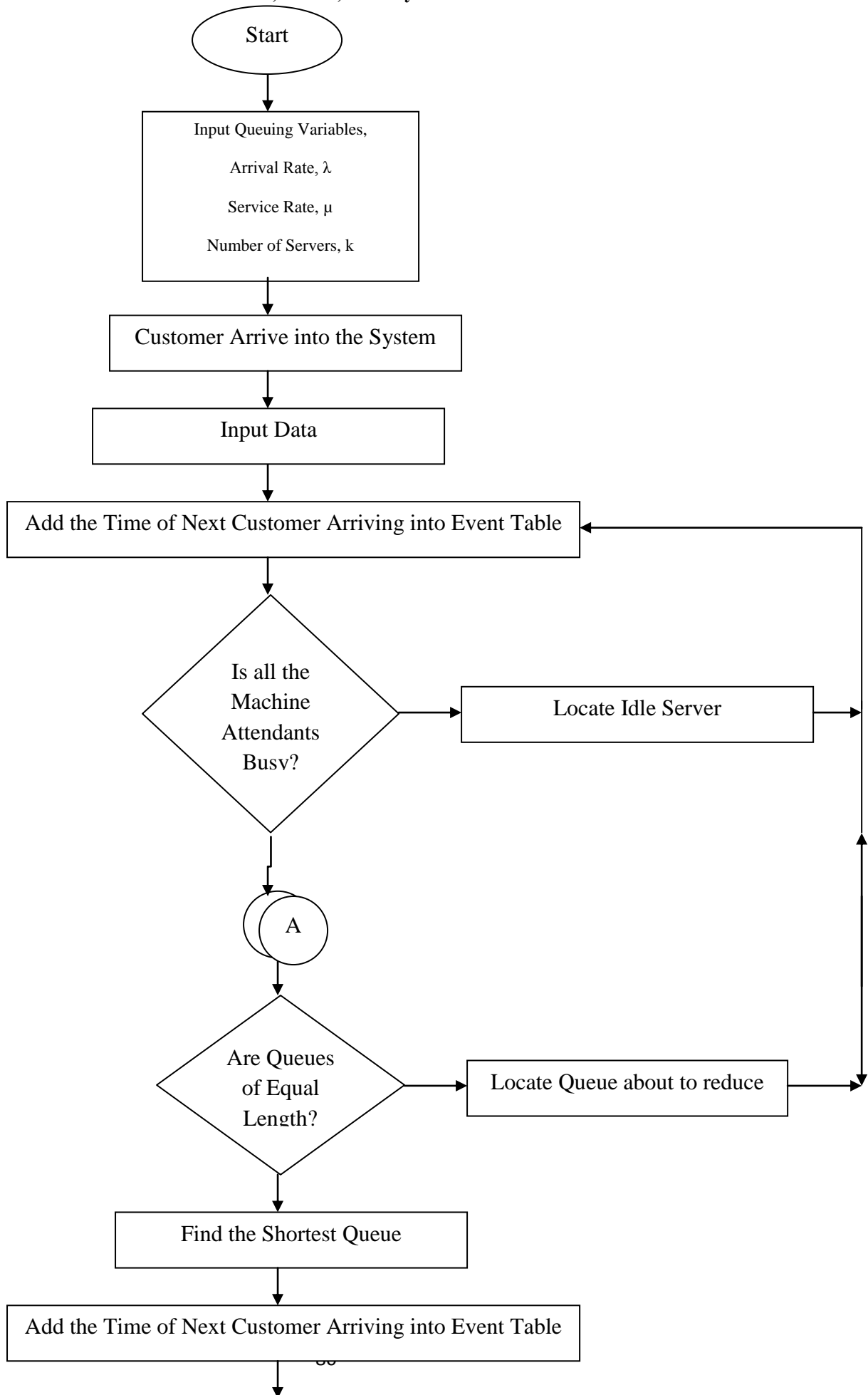
[8] H. Halabian, I. Lambadaris and C.H Lung: "Network capacity region of multi-queue multi server queuing system with time varying connectivity's", pp.1803-1807. ISIT 2010,

[9] T.B Sarah and A.H Lisa: "Development of a Random Sampling Procedure for Local Road Traffic Count Locations. Indiana: In: Bureau of transportation Statistic JWA/HMB, Mar. 2004.

[10] K. Sharma: "Operation Research (Theory and Application), Queuing System and Simulation, 4th Edition, 2004.

[11] R. Sidaway: "Study of Petrol Stations in Rural Scotland. General Agricultural Policy and Rural Development, Aug. 1998.

[12] D.D Yuncheng and S. Liang: "System Simulation and Application. Beijing, Mechanism Industry Press, 2002.



Jump out of the Loop

Delete the Customer Served from Event Table

Compute the Output  
 Average Waiting Time in the System,  $W_s$   
 Average Customer in the System,  $L_s$   
 Average Waiting Time in the Queue,  $W_q$   
 Average Customer in the Queue,  $L_q$   
 Probability of System Busy,  $P$   
 Probability of System Idle,  $P_0$

Compute Equation for  $W_s, L_s, W_q, L_q, P$  and  $P_0$

$$W_s = W_q + \frac{1}{\mu} = \frac{L_q}{\lambda} + \frac{1}{\mu}$$

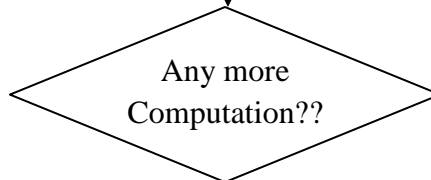
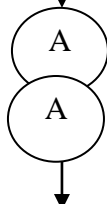
$$W_q = \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu}{(s\mu - \lambda)^2} \right] P_0 = \frac{L_q}{\lambda}$$

$$P_n = \begin{cases} \frac{\rho^n}{n!} P_0 & n \leq s \\ \frac{\rho^n}{s! s^{n-s}} P_0 & ; n > s; \rho = \lambda/s\mu \end{cases}$$

$$L_s = L_q + \frac{\lambda}{\mu}$$

$$L_q = \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\lambda\mu}{(s\mu - \lambda)^2} \right] P_0$$

$$P_0 = \left[ \sum_{n=0}^{s-1} \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n + \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \frac{s\mu}{s\mu - \lambda} \right]^{-1}$$



Yes

No

Print Result  
 $W_s, L_s, W_q, L_q, P$  and  $P_0$

(i) Compute the stability of the system  
 (ii) Check to know

